Using indicators of ecological stability in stochastic programming

Michal Houda[1]

Abstract. When building bigger constructions (transport, industrial, etc.), the EU law impose the so-called The EIA (Environmental Impact Assessment) process – evaluation of possible influences of the construction. All important constructions have to go through the process, prescribing obligatory the judgement of a series of factors that influence the environment and population health, grouped into several categories. Outputs of the EIA process are usually a set of recommendations and obligations to the investors and building companies in order to compensate the negative impacts of the constructions by additional arrangements.

In modelling phase, several uncertain parameters (e.g. future transport intensities) enter into the problem. Also many criteria of the EIA process are of subjective character, so modelling through utility functions (again not well defined) is proposed. In our contribution we develop an innovative approach to model the expenses devoted to obey the EIA rules by stochastic programming tools: especially, we represent uncertainty in parameters by their probabilistic distributions, and subjective utility function representing the ecological demands is modelled via so-called indicators of ecological stability. The model takes into account budget limitations, several legislative obligations (as pollution limits), and other ecological aspects influencing costs of transport constructions; the goal is to help choose, among possible compensating constructions and arrangements, the optimal ones. The resulting stochastic programming model could be seen as parallel to the usual V@R problem – type of problems used in finance.

Keywords: EIA process, indicator of ecological stability, stochastic programming, value-at-risk models.

JEL classification: C44
AMS classification: 90C15

1 Introduction

The growth of economics in whole has many visible et invisible manifestation in our surroundings, more or less distant. One of the very visible manifestation of these is number of constructions that grow every day at our nearness. This is not new phenomenon, at all; in fact it is old as the humanity itself. What is modern, in this context, is very strong emphasis on the impact of such development to the *environment*. Any of new big constructions, as transport line constructions (highways, railways), industrial or commercial areas, or other important constructions, cannot be nowadays realized without precise treatment of impacts (negative as positive) to the environment.

In [3] we already presented the whole legal framework of this treatment, let only briefly summarize it. By European Union law, every new construction has to pass through the process evaluating the impact of the construction to the environment. The process is divided into several phases, the most important one is so-called Environmental Impact Assessment (EIA). The output of this phase of the process is to evaluate if the negative impacts of the construction are still acceptable, in view of the society development, and (what is especially important in view of our present paper), to propose possible duties and arrangements to compensate these negative impacts. Unfortunately, the impacts are of very different nature and their evaluation is not simple at all. In [3], we already provided a description of the EIA categorization

[1] Institute of Information Theory and Automation of the ASCR, Pod Vodárenskou věží 4, 182 08 Praha 8, Czech Republic, e-mail: houda@ef.jcu.cz

of possible impacts (see also [1]), so we will not go through it again, only summarize that it includes human healthy impacts (air pollution, noise pollution, social-economic factors) and environmental impacts (climate, water, landscape, natural resources, ecosystems, etc.).

Our present intention is to continue research already started therein: to develop stochastic programming models incorporating to the above decision process some uncertainty factors, as unknown transport intensities (on transport line constructions), efficiency of compensation, or subjectivity of some EIA criteria.

2 Model description

Denote $x \in X \subset \mathbb{R}^n$ the collection of the possible compensations and arrangements ready to be used in some construction. The values of the variable x can be discrete or continuous according to the nature of the arrangement. Let $\xi \in \Xi$ be the random vector representing uncertainty factors, with $\Xi \subset \mathbb{R}^s$ being the predefined support of ξ. From the computational perspective, it is necessary to consider the probability distribution of ξ to be known in advance.

The actual expenses of all arrangements are represented by the cost function $c : X \times \Xi \to \mathbb{R} : (x; \xi) \mapsto c(x; \xi)$. We make here the first simplification and suppose c to be a linear function, i. e., $c(x; \xi) = c^T x$ where $c \in \mathbb{R}^n$ are constant unit costs of the arrangements. Let B be the budget limit on these expenses.

All factors of subjective and evaluative character are represented by the utility function $u : X \times \Xi : (x; \xi) \mapsto u(x; \xi)$. The quantification of this function is difficult; later, we will use a function based on the indicators of ecological stability to represent the values of the function u.

We are know ready to formulate the optimization model with uncertainty. Instead of viewing the problem as (traditionally) cost-minimizing, we use a utility-maximizing view:

$$\text{maximize } u(x; \xi) \text{ subject to } c^T x \leq B, x \in X_0. \tag{1}$$

where $X_0 \subset X$ includes all deterministic constraints, technical parameters, and 0-1 type constraints. This is an optimization model with unknown uncertainty parameters, so we are looking for a probabilistic formulation incorporating available information on the random parameter ξ. Before do that we deal first with subjective utility function u. Similarly as in [3] we use so-called indicators of ecological stability.

Indicators of ecological stability is a set of well defined and quantified variables that evaluates the quality of environment. The overal set of these indicators is still growing (especially in case of indicators qualifying such things as landscape, efficiency, total prosperity of environment, and similar); the actual standard set, and propositions to new possible indicators are at the disposition of the reader on the homepage of the European Environmental Agency (EEA, see [2]).

3 β-V@R formulation of the problem

The main idea of our approach is to replace the subjective utility function by a weighted sum of objective indicators of ecological stability. Let $g : \mathbb{R}^n \times \Xi \to \mathbb{R}^G : (x; \xi) \mapsto g(x; \xi)$ be a function representing the values of the EEA indicators. Moreover, we introduce a new parameter L representing a required limit on values of g, and the weights $w \in [0; 1]^G$ representing importance of each of indicators. Now we are ready to incorporate the probabilistic information on ξ provided by its probability distribution. We do it through imposing the probabilistic constraint in the form

$$\Pr\{w^T g(x; \xi) \geq L\} \geq \beta \tag{2}$$

where L is now considered as the decision variable, and $\beta \in (0; 1)$ is a prescribed (sufficiently high) value of probability of exceeding the limit L.

The whole proposed formulation of the problem read:

$$\text{maximize } L \text{ subject to } \Pr\{w^T g(x; \xi) \geq L\} \geq \beta, c^T x \leq B, x \in X_0. \tag{3}$$

The formulation is seen parallel to the known β-V@R problems, thoroughly analyzed in finance optimization (see e. g. [4] and references therein). The only difference is that our left-hand side of probabilistic constraint does not represent losses (as in original formulation) but the positive effect of arrangements. In fact, this is not any real obstacle, and we can use without worries the methods developed by many authors for V@R problems.

3.1 Example

Consider an artificial example of building a segment of a (non-specified) highway. For simplicity we consider two indicators of ecological stability only:

- $1 - i_1 \ldots$ excedance of the air pollution limits (percents of area where the limits are exceeded);
- $-i_2 \ldots$ noise pollution (number of habitants exposed to heavy noise).

To share previously used notation we use this modified notation so that indicators i_1 and i_2 are represented by positive values.

Consider further three possible arrangements in mind to diminish the negative impacts of the building.

- $x_1 \ldots$ imposed speed limit of 80 km/h;
- $x_2 \ldots$ imposed speed limit of 110 km/h;
- $x_3 \ldots$ noise wall dimensions.

The variables x_1 and x_2 are binary (the speed limit is imposed or not), the variable x_3 is continuous and could represent the length of noise wall (with standard width and height), or the consumption of the material to build it.

Finally, the only random factor considered in the example is ξ, random transport intensity on the highway segment considered. Assume that the values of indicators depends linearly on x and ξ. (In fact, we expect that this is far from reality for real problems and is subject to the future research.) Then we could write

$$\begin{aligned} i_1 &= (a_{10} + a_{11}x_1 + a_{12}x_2)\xi \\ i_2 &= (a_{20} + a_{21}x_1 + a_{22}x_2 + a_{23}x_3)\xi \end{aligned}$$

The parameters a_{11} and a_{12} represent the positive effect of the imposed speed limit to the air pollution; a_{21} and a_{22} the same to the noise pollution. Denote $x = (1, x_1, x_2)$, $a_0 = (a_{10}, a_{20})$, $A = (a_{ij})_{i,j=1,2}$. Consider further that the overall impact of the highway construction is measured by the weighted sums of indicators i_1 and i_2 with $w = (w_1, w_2)$ beeing the weights of the indicators. In such notation, the β-V@r model formulation of the problem reads:

$$\text{maximize } L \tag{4}$$

subject to

$$\begin{aligned} \Pr\big\{w_1 i_1 + w_2 i_2 = w^T(a_0 + Ax)\xi \geq L\big\} &\geq \beta \\ c^T x &\leq B \\ x_1 + x_2 &\leq 1 \\ x_{1,2} \in \{0,1\}, x_3 &\geq 0 \end{aligned}$$

The resulting optimization program is the linear stochastic programming mixed-integer problem with probabilistic constraints, and it is solvable under some additional assumption on distribution of the random variable.

4 β-CV@R formulation

Inspired by [4] we now proceed to better formulation of the model. The β-CV@R model is, according to [4] formulated as the conditional expectation of losses above the amount L. Sharing the notation of the previous section, the β-CV@R formulation of our optimization problems reads

$$\text{to find } \phi_\beta(x) := \frac{1}{1-\beta}\,\mathbb{E}\big[w^T g(x;\xi) \mid w^T g(x;\xi) \le L_\beta(x)\big] \tag{5}$$

where

$$L_\beta(x) = \max\big\{L : \Pr\{w^T g(x;\xi) \ge L\} \ge \beta, c^T x \le B, x \in X_0\big\} \tag{6}$$

i. e., $L_\beta(x)$ is the optimal value of the β-V@R formulation of the problem.

Denote

$$F_\beta(x;L) = -L + \frac{1}{1-\beta}\,\mathbb{E}[L - w^T g(x;\xi)]^+ \tag{7}$$

where $[t]^+$ is positive part of t (i. e., $[t]^+ = t$ if $t \ge 0$ and zero otherwise).

Proposition 1. *As a function of L, $F_\beta(x;L)$ is convex, continuously differentiable and β-CV@R value ϕ_β of any $x \in X$ can be determined from the formula*

$$\phi_\beta(x) = \min_L F_\beta(x;L). \tag{8}$$

Proof. The proposition follows directly from Theorem 1 in [4]. □

The proposition enable simple numerical scheme to solve the β-CV@R formulation of our problem because the minimization of the convex differentiable function is numerically easy. Moreover, the explicit dependence on β-V@R value L_β (which computation may not be always easy) is discarded. We finally illustrate the new formulation on Example 3.1.

4.1 Example continued

The β-CV@R formulation of our example problem reads simply

$$\phi_\beta(x) = \frac{1}{1-\beta}\,\mathbb{E}\big[w^T(a_0 + Ax)\xi \mid w^T(a_0 + Ax)\xi \le L_\beta(x)\big] \tag{9}$$

with L_β being the optimal value of the problem (4). Using (7) and Proposition 1 we arrive at final problem formulation:

$$\text{minimize } -L + \frac{1}{1-\beta}\,\mathbb{E}[L - w^T(a_0 + Ax)\xi]^+ \tag{10}$$

subject to

$$\begin{aligned} c^T x \le B, &\quad x_1 + x_2 \le 1 \\ L \in \mathbb{R}, &\quad x_{1,2} \in \{0,1\}, x_3 \ge 0 \end{aligned}$$

If we have an approximation or estimate of the vector ξ at our hand (for example, done by sampling), various numerical techniques can be used to estimate value and solution of problem (10).

5 Conclusion

In this paper we presented a V@R motivated approach to solve problems coming from the environmental politics. We designed a probabilistic approach to model uncertainties and subjective factors in evaluating impacts of constructions to the environment, incorporating well-defined and objectively determined indicators of ecological stability. Our model are especially useful in that they obey assumptions on traditional financial V@R and CV@R models, so that open the door to many modelling and computational methods developed originally for these financial models.

Acknowledgements

This research was partially supported by the Czech Science Foundation under the project No. P402/10/0956.

References

[1] Council directive 85/337/EEC of 27 June 1985 on the assessment of the effects of certain public and private projects on the environment. *Official Journal of the European Union*, L 175, 05/07/1985:40–48, June 1985.

[2] EEA Core set of indicators [online]. European Environment Agency. Available from World Wide Web: <http://themes.eea.europa.eu/IMS/CSI>, February 2011 [cited 20 May 2011].

[3] Houda, M.: Environmental factors in optimization of traffic line construction expenses. In *Proceedings of the 24th International Conference Mathematical Methods in Economics 2010* (M. Houda and J. Friebelová, eds.), University of South Bohemia in Česke Budějovice, 2010, 262–267.

[4] Rockafellar, R. T., and Uryasev, S.: Optimization of conditional value-at-risk. *The Journal of Risk* **2** (2000), 21–41.